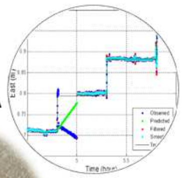
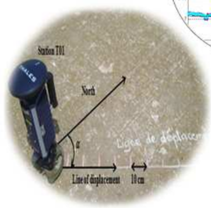
Centre des Techniques Spatiales

Analysis of GPS coordinates time series by Kalman filter (7544)



Bachir GOURINE, Abdelhalim NIATI, Achour BENYAHIA and Mokhfi BRAHIMI



Contact:
Dr. Bachir Gourine (*Researcher*)
bachirgourine@yahoo.com
Division of Space Geodesy
Centre of Space Techniques (CTS)
Arzew – ALGERIA.

FIG Working Week
17 - 21 May, Bulgaria
From the wisdom of the ages to the challenges of modern world
SOFIA 2015

0 Presentation Plan

1 Introduction

- Kalman filter is an appropriate tool for analyzing time series of position when the deformations are modeled as a linear dynamic system. Kalman filter gives the best estimate.
- The Kalman filter has numerous applications in technology : guidance, navigation and control of vehicles, particularly aircraft and spacecraft, time series analysis used in fields such as signal processing and econometrics…
- Two dynamic systems are chosen to describe the same dynamic of the deformation: first, the identity model when position follows a Random Walk Process (RWP) and second, the kinematic model when velocity follows a RWP.
- **Objective:** application of the Kalman Filter in position time series analysis : filtering, prediction & smoothing

3

2 Kalman Filter analysis

KF Formulation

Continuous-time

$\dot{X} = F(t).X(t) + G(t).w(t)$ Dynamic system

$\tilde{Z} = H(t).X(t) + v(t)$ Measurement system

Discretised-time

$X_k = \Phi_{k-1}X_{k-1} + \Gamma_{k-1}w_{k-1}$ Dynamic system

$\tilde{Z}_k = H_k.X_k + v_k$ Measurement system

Gaussian, White, Zero-mean

Var (w) = Qd_{k-1}
Var (v) = R_k
E (v.w) = 0

- **Initial conditions :**

$E[x_0] = \hat{x}_0$

$E[(x_0 - \hat{x}_0).(x_0 - \hat{x}_0)^T] = P(0)$

2 Kalman Filter analysis

Dynamic systems : ➢ Qualify the dynamical behavior of stations / **02 models** :

Identity Model

Station position follows a Random Walk Process noise (RWP) $\dot{\xi} = w$

$$X_k = \Phi_{k-1} X_{k-1} + \Gamma_{k-1} w_{k-1}$$

$$\begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix}_k = I_3 . \begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix}_{k-1} + I_3 . \Delta t_k \begin{bmatrix} w_E \\ w_N \\ w_U \end{bmatrix}_{k-1}$$

$$\tilde{Z}_k = H . X_k + \eta_k$$

$$\begin{bmatrix} \delta \tilde{E} \\ \delta \tilde{N} \\ \delta \tilde{U} \end{bmatrix}_k = I_3 . \begin{bmatrix} \delta E \\ \delta N \\ \delta U \end{bmatrix}_k + \begin{bmatrix} \eta_E \\ \eta_N \\ \eta_U \end{bmatrix}_k$$

Kinematic Model

Station Velocity follows a RWP $\ddot{\xi} = w$

$$X_k = \Phi_{k-1} X_{k-1} + \Gamma_{k-1} w_{k-1}$$

$$\begin{bmatrix} \delta E \\ \delta N \\ \delta U \\ v_E \\ v_N \\ v_U \end{bmatrix}_k = \begin{bmatrix} I_3 & I_3 \Delta t_k \\ 0_{3,3} & I_3 \end{bmatrix} \begin{bmatrix} \delta E \\ \delta N \\ \delta U \\ v_E \\ v_N \\ v_U \end{bmatrix}_{k-1} + \begin{bmatrix} I_3 \Delta t_k^2/2 \\ I_3 \Delta t_k \end{bmatrix} \begin{bmatrix} w_E \\ w_N \\ w_U \end{bmatrix}_{k-1}$$

$$\tilde{Z}_k = H . X_k + \eta_k$$

$$\begin{bmatrix} \delta \tilde{E} \\ \delta \tilde{N} \\ \delta \tilde{U} \end{bmatrix}_k = [I_3 0_{3,3}] \begin{bmatrix} \delta E \\ \delta N \\ \delta U \\ v_E \\ v_N \\ v_U \end{bmatrix}_k + \begin{bmatrix} \eta_E \\ \eta_N \\ \eta_U \end{bmatrix}_k$$

2 Kalman Filter analysis

KalmanFilteris a state estimator with a minimum mean-squared error [Ribeiro 2004], used for : filtering, prediction and smoothing.

- **Filtering:**The classical Kalman filter was first established by Rudolph E. Kalman in his influential paper [Kalman 1960]. The filtered estimate of X_k only takes into account the past information relative to X_k [Pieter 2008]. The Kalman filter is applicable in real time. The algorithm is divided in two steps: prediction step and measurement update step.
- **Prediction:**when a measurement is missing or qualified as faulty or erroneous the corresponding update measurement step in the precedent algorithm must not occur [Farrell 2008]. In our case, erroneous data are present in some epochs then it is possible to predict states at such epochs
- **Smoothing:** The smoother is called RTS since its implementation was derived by H. Rauch, K. Tung and C. Striebel in 1965 [Grewal 2001].By incorporating the past and future observations relative to X_k, we can obtain a more refined state estimate [Abeel 2008]. For this effect, smoothing can be done only in post-processing.

6

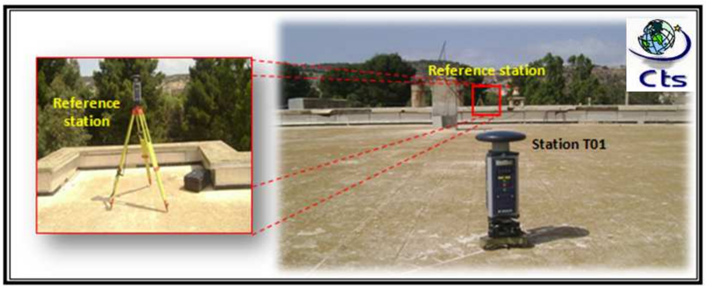
3

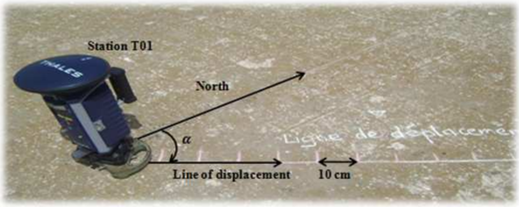
Application

FIG SOFIA 2015

Experience

- 02 ZMAX GPS receivers
- Short Distance of **45 m**
- Sample time : **1s**
- Obs duration: **09h**
- Date: **13th May 2014 at 11h**
- Place: Roof of CTS building





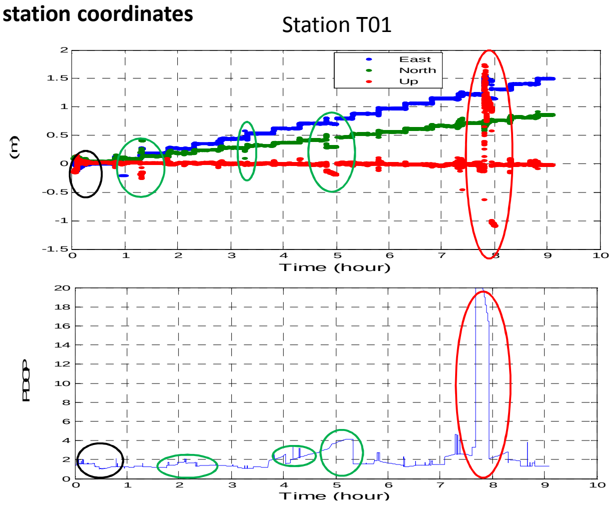
7

3

Application

FIG SOFIA 2015

Time series of station coordinates

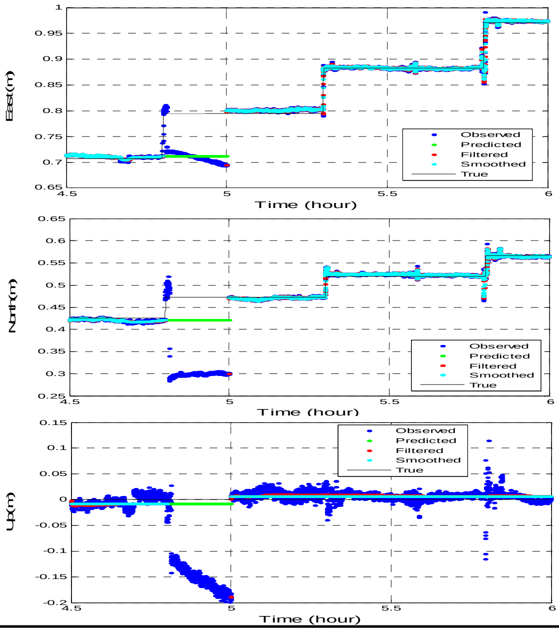


8

3 **Application**

FIG SOFIA 2015

Kalman filter with Identity model

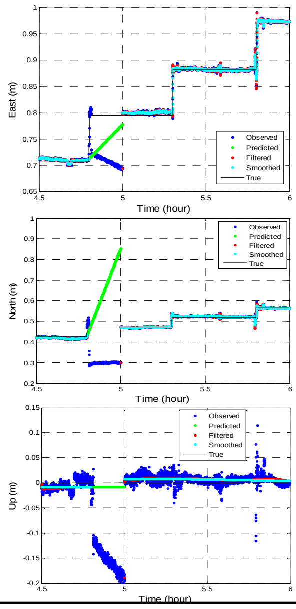


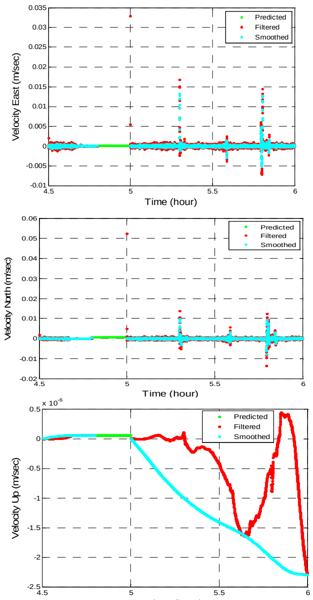
3 **Application**

FIG SOFIA 2015

Kalman filter with kinematic model

Position





Velocity

10

3 Application

Results

Average of standard deviations	Observed	Identity model				Kinematic model			
		Filtered	Imp (%)	Smoothed	Imp (%)	Filtered	Imp (%)	Smoothed	Imp (%)
$\bar{\sigma}_E$ (mm)	30.2	10.45	65.4	8.31	72.5	14.08	53.4	7.80	74.2
$\bar{\sigma}_N$ (mm)	31.6	6.73	78.7	5.29	83.2	11.39	74	6.14	80.6
$\bar{\sigma}_U$ (mm)	74.8	0.65	99.1	0.28	99.6	0.73	99	0.34	99.5

1/ Better precision of filtered and smoothed times series vs. observed time series.

2/ Smoothing gives more accurate coordinates than the filtered coordinates for both models

3/ Identity model seems to perform better than the kinematic model (for North and Up components).

4/ RMS of Up component is improved with a great amount of about (99% to 99.6%) : Up component was invariant over time (9h), so it is well predicted by the two dynamic systems.

5/ East RMS (30.2 mm) is slightly accurate than North RMS (31.6 mm), Bust processed East component is less accurate than the processed North one : due to azimuth angle α =62° → most displacements of the receiver happen on the East axis and thus the noise.

11

4 Conclusion and perspectives

- The identity model seems to be the more adaquete model for describing the motion of the receiver than the kinematic one.
- The prediction precision decreases as the time of measurement lack increases.
- Compared to the filtering, smoothing provides more accurate solution of about (72.5%, 83.2%, 99.6%) and (74.2%, 80.6%, 99.5%), according to local coordinates (E, N, U), for identity and kinematic models, respectively.

As perspectives ,

- The correlations between the three observed time series should be investigated and taken into account in the construction of the measurement covariance matrix.
- The GPS coordinate time series are often correlated with time, so the white noise assumption of the noise is not justified, consequently a shaping filter should be appended to the dynamic system to reduce this correlation.
- Establishing a mechanism that enable -at a certain level- detection of anomalies caused by bad geometry or some error sources inherent in relative GPS observations.

12

Thank you for your attention